



**Answer SIX questions from Section A and THREE questions from Section B.**

The numbers in square brackets in the right-hand margin indicate the provisional allocations of maximum marks per sub-section of a question.

$$\text{Boltzmann's constant } k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

$$\text{The gas constant } R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$\text{Avogadro's number } N_A = 6.02 \times 10^{23}$$

$$\text{Acceleration due to gravity } g = 9.81 \text{ m s}^{-1}$$

$$\text{Freezing point of water } 0^\circ\text{C} = 273.15\text{K}$$

**SECTION A**

[part marks]

1. Explain what is meant by an *ideal gas* in classical thermodynamics. Write down the ideal gas equation of state, and explain the meaning of any symbols used. [3]

A water manometer is used to measure the pressure of an ideal gas at 280 K. Calculate the pressure if the gas can support a column of water 11 m high. The density of the water is  $1.0 \times 10^3 \text{ kg m}^{-3}$ . [2]

With reference to an ideal gas, explain what is meant by the *absolute zero* of temperature. [2]

2. State the assumptions underlying the kinetic theory of gases, and derive the relationship; [7]

$$pV = \frac{1}{3} N_A m \overline{v^2}.$$

Explain the meaning of all symbols used.

3. 1 mole of liquid helium at its boiling point of 4.2 K is isothermally converted to gas, at a constant pressure of  $1 \times 10^5 \text{ Pa}$ . The latent heat of evaporation for liquid helium is  $83.6 \text{ J mol}^{-1}$ , and the molar volume of the liquid helium is  $3.33 \times 10^{-5} \text{ m}^3 \text{ mol}^{-1}$ . If the helium gas is treated as ideal, calculate;
- (a) the work done by the helium, [3]
- (b) the heat supplied to the helium, [2]
- (c) the change in internal energy of the helium. [2]

4. Thermal conduction is governed by the equation;

$$\frac{Q}{\Delta t} = \kappa A \frac{\Delta T}{\Delta x}.$$

Explain in words the meaning of this equation. [3]

A pond has surface area  $100 \text{ m}^2$ , and is covered by a layer of ice that is  $0.01 \text{ m}$  thick. The top surface of the ice is held at  $-10^\circ\text{C}$  by the cold air above, and the water immediately beneath the ice is at  $0^\circ\text{C}$ . Calculate the power lost from the water through the ice by thermal conduction. The thermal conductivity of ice is  $2.0 \text{ W m}^{-1} \text{ K}^{-1}$ .

[2]

At what rate is ice being formed at the ice-water interface if the latent heat of fusion of water is  $3.33 \times 10^5 \text{ J kg}^{-1}$ ?

[2]

5. Write down a statement of the *Second Law of Thermodynamics*. [2]

A  $10 \text{ g}$  cube of ice at  $0^\circ\text{C}$  is placed into  $100 \text{ g}$  of water at  $20^\circ\text{C}$ . The system is then isolated and allowed to reach equilibrium. Calculate;

(a) The final temperature, [2]

(b) The total entropy change. [3]

The heat capacity of water is  $4.19 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$ , and the latent heat of melting of ice is  $3.33 \times 10^5 \text{ J kg}^{-1}$ . You may assume that there is no volume change.

6. Show that the entropy change of an ideal gas undergoing a reversible change of state can be written: [4]

$$\Delta S = nC_v \ln\left(\frac{T_2}{T_1}\right) + nR \ln\left(\frac{V_2}{V_1}\right).$$

Use this equation to calculate the entropy change when  $1 \text{ mole}$  of He, at  $280 \text{ K}$  and occupying a volume  $0.02 \text{ m}^3$ , mixes with  $1 \text{ mole}$  of Ar, at  $320 \text{ K}$  and occupying  $0.02 \text{ m}^3$ . Both He and Ar can be treated as ideal monatomic gases. [3]

7. The van der Waals equation of state can be written

$$\left(p + \frac{N^2 a}{V^2}\right)(V - Nb) = NkT.$$

Briefly describe the purpose of this equation, and the conditions under which you would expect it to be valid. [4]

Explain the physical interpretation of the constants  $a$  and  $b$ , with reference to the ideal gas equation of state and the interparticle interactions. [3]

8. Draw the face-centred and body-centred cubic crystal structures. In each case, state whether the atoms are close-packed, and give the coordination number. [5]

Give a statement of the *Third Law of Thermodynamics* that describes the low temperature limits of the entropy and heat capacity of a crystal. [2]

## SECTION B

[part marks]

9. Write down a statement of the *First Law of Thermodynamics*. Explain any symbols that you use. [3]

Show that for a mole of ideal gas the difference in molar heat capacity at constant pressure and constant volume is given by [4]

$$C_p - C_v = R.$$

With reference to simple gases, explain what is meant by a *degree of freedom* and the *equipartition of energy*. [4]

Sketch  $C_v$  as a function of temperature for a mole of a simple diatomic gas, such as  $H_2$ , and briefly explain its qualitative behaviour. [6]

A mole of ideal monatomic gas is at 300 K,  $1.5 \times 10^5$  Pa and  $0.010 \text{ m}^3$ . Calculate the change in internal energy of the gas if it is heated reversibly to 350 K at; [3]

- (a) constant volume,  
(b) constant pressure.

10. Derive the adiabatic equation of state for an ideal gas; [5]

$$pV^\gamma = \text{constant}.$$

Explain the physical significance of the constant  $\gamma$  in this equation. [3]

An ideal gas at a pressure  $p_1$  and volume  $V_1$  expands reversibly to a volume  $V_2$ . Sketch this process on a  $p$ - $V$  diagram under the following conditions; [4]

- (a) isothermal expansion, monatomic gas,  
(b) adiabatic expansion, monatomic gas,  
(c) adiabatic expansion, diatomic gas.

Show that the work done by an ideal gas in a reversible adiabatic expansion can be written; [4]

$$W = \frac{p_1 V_1}{\gamma - 1} \left[ 1 - \left( \frac{V_1}{V_2} \right)^{\gamma - 1} \right].$$

Calculate the work done by a mole of monatomic gas if it undergoes a reversible adiabatic expansion from  $p_1 = 2 \times 10^5$  Pa and  $V_1 = 0.01 \text{ m}^3$  to  $V_2 = 0.02 \text{ m}^3$ . Calculate also the final temperature. [4]

11. The Maxwell-Boltzmann velocity distribution in one dimension can be written

$$P(v_x)dv_x = \left(\frac{m}{2\pi kT}\right)^{1/2} \exp\left(\frac{-mv_x^2}{2kT}\right)dv_x.$$

Explain briefly what this function represents, and its physical origins. For what value of  $v_x$  does  $P(v_x)dv_x$  reach a maximum? [4]

Derive the Maxwell-Boltzmann speed distribution from the above one dimensional velocity distribution. [4]

Using the equipartition of energy, show that the root mean square speed of the Maxwell-Boltzmann distribution is given by [2]

$$v_{rms} = \sqrt{\frac{3kT}{m}}.$$

Oxygen molecules have a molar mass  $0.016 \text{ kg mol}^{-1}$ . Calculate the root mean square speed of an oxygen molecule in the gas at 300 K and 600 K. [2]

Sketch the Maxwell-Boltzmann speed distributions for oxygen molecules at 300 K and 600 K. In each case, mark the root mean square speed. [5]

If the partial pressure of oxygen in the atmosphere is  $0.25 \times 10^5 \text{ Pa}$  at sea-level, calculate the partial pressure of oxygen at a height of 2000 m above sea-level. You may assume that the atmosphere is ideal and isothermal, with  $T = 300 \text{ K}$ . [3]

12. Sketch an ideal reversible Carnot cycle operating between two temperatures  $T_H > T_C$  on a  $p$ - $V$  and a  $T$ - $S$  diagram. Label the processes involved, and indicate the heat flow and work done. [8]

Calculate the heat and entropy changes of the above cycle. Explain then what is meant by a *state function* and an *irreversible process*. [8]

An ideal reversible Carnot cycle operates between 270K and 300K. Calculate the coefficient of performance if it is used as [4]

- (a) a heat pump being used in heating mode,
- (b) a refrigerator.

13. The Mie interaction potential can be written:

$$u(r) = \frac{A}{r^n} - \frac{B}{r^m}.$$

Explain briefly what this function represents, and its physical origins. [4]

Obtain an expression for the force,  $F(r)$ , between two particles interacting through the Mie potential. Show then that the equilibrium separation between two particles interacting through the Mie potential is given by [5]

$$r_0 = \left( \frac{mB}{nA} \right)^{\frac{1}{m-n}}.$$

Sketch on the same graph the Mie interaction potential and force between two particles. Mark the binding energy and the equilibrium spacing,  $r_0$ , on your diagram. [6]

Explain briefly what values of  $m$  and  $n$  you would choose in the Mie function to represent the interaction potential between the following particles; [5]

- (a) a sodium and a chloride ion ( $\text{Na}^+$  and  $\text{Cl}^-$ ),
- (b) two argon atoms ( $\text{Ar}$  and  $\text{Ar}$ ).