Solution to Exercise 7.1.

For opacity law $\kappa = \kappa_0 \rho^{\lambda} T^{-\nu}$ $\kappa = \kappa_0 \rho^{\lambda} T^{-\nu}$ $\kappa = \kappa_0 \rho^{\lambda} T^{-\nu}$ (equation 7.24), equation of radiative energy transfer gives (see equation 7.25)

$$L \propto R^{3\lambda-\nu} M^{3+\nu-\lambda} .$$

$L \propto R^{3\lambda-\nu} M^{3+\nu-\lambda}$ $L \propto R^{3\lambda-\nu} M^{3+\nu-\lambda}$.

In our case $\lambda=0$, $\nu=0$, and we get

$$L \propto M^3 .$$

$L \propto M^3$ $L \propto M^3$.

For $\varepsilon \propto \rho T^{\alpha}$ $\varepsilon \propto \rho T^{\alpha}$$\varepsilon \propto \rho T^{\alpha}$, energy generation equation gives (see equation 6.10)

$$L \propto \frac{M^{2+\alpha}}{R^{3+\alpha}} .$$

$L \propto \frac{M^{2+\alpha}}{R^{3+\alpha}}$ $L \propto \frac{M^{2+\alpha}}{R^{3+\alpha}}$.

In our case $\alpha = 18$ $\alpha = 18$$\alpha = 18$, and we get

$$L \propto M^{20} R^{-21} .$$

$L \propto M^{20} R^{-21}$ $L \propto M^{20} R^{-21}$.

We thus have $M^{20} R^{-21} \propto M^3$ $M^{20} R^{-21} \propto M^3$ $M^{20} R^{-21} \propto M^3$, and hence

$$R \propto M^{17/21} .$$

Solution to Exercise 7.2.

For opacity law $\kappa = \kappa_0 \rho^{\lambda} T^{-\nu}$ $\kappa = \kappa_0 \rho^{\lambda} T^{-\nu}$ $\kappa = \kappa_0 \rho^{\lambda} T^{-\nu}$ (equation 7.24), equation of radiative energy transfer gives (see equation 7.25)

$$L \propto R^{3\lambda-\nu} M^{3+\nu-\lambda} .$$

$L \propto R^{3\lambda-\nu} M^{3+\nu-\lambda}$ $L \propto R^{3\lambda-\nu} M^{3+\nu-\lambda}$.

Now we have $\lambda=1$, $\nu=3.5$, and we get

$$L \propto M^{5.5}R^{-0.5}.$$

$$L \propto M^{5.5}R^{-0.5}.$$
$$L \propto M^{5.5}R^{-0.5}.$$

For $\varepsilon \propto \rho T^{\alpha}$ $\varepsilon \propto \rho T^{\alpha}$ $\varepsilon \propto \rho T^{\alpha}$, the energy generation equation gives (see equation 6.10)

$$L \propto \frac{M^{2+\alpha}}{R^{3+\alpha}}.$$

$$L \propto \frac{M^{2+\alpha}}{R^{3+\alpha}} \quad L \propto \frac{M^{2+\alpha}}{R^{3+\alpha}}.$$

Now we have $\alpha = 18$ $\alpha = 18$ $\alpha = 18$, and we get

$$L \propto M^{18}R^{-19}.$$

$$L \propto M^{18}R^{-19} \quad L \propto M^{18}R^{-19}.$$

We thus have $M^{18}R^{-19} \propto M^{5.5}R^{-0.5}$ $M^{18}R^{-19} \propto M^{5.5}R^{-0.5}$ $M^{18}R^{-19} \propto M^{5.5}R^{-0.5}$, and hence

$$R \propto M^{25/37}.$$

$$R \propto M^{25/37} \quad R \propto M^{25/37}.$$

With this relation between mass and radius, we have

$$L \propto M^{\frac{191}{37}}.$$

$$L \propto M^{\frac{191}{37}} \quad L \propto M^{\frac{191}{37}}.$$

But $L \propto R^2T_{eff}^4$ $L \propto R^2T_{eff}^4$ $L \propto R^2T_{eff}^4$, and hence

$$M \propto T_{eff}^{\frac{4\times 37}{141}}$$

$$M \propto T_{eff}^{\frac{4\times 37}{141}} \quad M \propto T_{eff}^{\frac{4\times 37}{141}}$$

, which gives

$$L \propto T_{eff}^{5.4}.$$

$$L \propto T_{eff}^{5.4} \quad L \propto T_{eff}^{5.4}.$$

The slope of the line in $\log L - \log T_{eff}$ $\log L - \log T_{eff}$ $\log L - \log T_{eff}$ coordinates is thus about 5.4.