Solution to Exercise 3.1.

Introducing new independent variable

$$x^2 = \frac{mv^2}{2kT},$$

$$x^2 = \frac{mv^2}{2kT}, x^2 = \frac{mv^2}{2kT},$$

we have

$$\int_0^\infty f(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2kT}{m}\right)^{3/2}\int_0^\infty x^2 e^{-x^2}dx$$

$$= \frac{4}{\sqrt{\pi}}\int_0^\infty x^2 e^{-x^2}dx\,.$$

$$\int_0^\infty f(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2kT}{m}\right)^{3/2}\int_0^\infty x^2 e^{-x^2}dx$$

$$= \frac{4}{\sqrt{\pi}}\int_0^\infty x^2 e^{-x^2}dx\,.$$

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$$= \frac{4}{\sqrt{\pi}}\int_0^\infty x^2 e^{-x^2}dx\,.$$

Integrating by parts,

$$\int_0^\infty f(v)\,dv = -\frac{2}{\sqrt{\pi}}\int_0^\infty x\frac{d}{dx}\left(e^{-x^2}\right)dx$$

$$= \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-x^2}dx = 1.$$

$$\int_0^\infty f(v)\,dv = -\frac{2}{\sqrt{\pi}}\int_0^\infty x\frac{d}{dx}\left(e^{-x^2}\right)dx$$

$$= \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-x^2}dx = 1.$$

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$$= \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-x^2}dx = 1.$$

Using the same independent variable,

$$\left\langle \frac{mv^2}{2} \right\rangle = \int_0^\infty \frac{mv^2}{2} f(v)\,dv$$

$$= \frac{m}{2}4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2kT}{m}\right)^{5/2}\int_0^\infty x^4 e^{-x^2}dx$$

$$= \frac{4}{\sqrt{\pi}}kT\int_0^\infty x^4 e^{-x^2}dx\,.$$

$$\left\langle \frac{mv^2}{2} \right\rangle = \int_0^\infty \frac{mv^2}{2} f(v)\,dv$$

$$= \frac{m}{2} 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} \left(\frac{2kT}{m} \right)^{5/2} \int_0^\infty x^4 e^{-x^2} dx$$

$$= \frac{4}{\sqrt{\pi}} kT \int_0^\infty x^4 e^{-x^2} dx \ .$$

$$\left\langle \frac{mv^2}{2} \right\rangle = \int_0^\infty \frac{mv^2}{2} f(v)\,dv$$

$$= \frac{m}{2} 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} \left(\frac{2kT}{m} \right)^{5/2} \int_0^\infty x^4 e^{-x^2} dx$$

$$= \frac{4}{\sqrt{\pi}} kT \int_0^\infty x^4 e^{-x^2} dx \ .$$

Integrating by parts,

$$\left\langle \frac{mv^2}{2} \right\rangle = -\frac{2}{\sqrt{\pi}} kT \int_0^\infty x^3 \frac{d}{dx}\left(e^{-x^2}\right) dx$$

$$= \frac{6}{\sqrt{\pi}} kT \int_0^\infty x^2 e^{-x^2} dx$$

$$= -\frac{3}{\sqrt{\pi}} kT \int_0^\infty x \frac{d}{dx}\left(e^{-x^2}\right) dx$$

$$= \frac{3}{\sqrt{\pi}} kT \int_0^\infty e^{-x^2} dx = \frac{3}{2} kT \, .$$

$$\left\langle \frac{mv^2}{2} \right\rangle = -\frac{2}{\sqrt{\pi}} kT \int_0^\infty x^3 \frac{d}{dx}\left(e^{-x^2}\right) dx$$

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$$= \frac{3}{\sqrt{\pi}} kT \int_0^\infty e^{-x^2} dx = \frac{3}{2} kT \, .$$

Solution to Exercise 3.2.

We have

$$\mu^{-1} = \sum_i X_i \frac{Z_i + 1}{A_i} = 2X + \frac{3}{4} Y$$
$$= 2X + \frac{3}{4}(1 - X) = \frac{5}{4} X + \frac{3}{4},$$

$$\mu^{-1} = \sum_i X_i \frac{Z_i + 1}{A_i} = 2X + \frac{3}{4} Y \qquad \mu^{-1} = \sum_i X_i \frac{Z_i + 1}{A_i} = 2X + \frac{3}{4} Y$$
$$= 2X + \frac{3}{4}(1 - X) = \frac{5}{4} X + \frac{3}{4}, \qquad = 2X + \frac{3}{4}(1 - X) = \frac{5}{4} X + \frac{3}{4},$$

hence

$$\mu = \frac{4}{3 + 5X}.$$

$$\mu = \frac{4}{3 + 5X} \quad \mu = \frac{4}{3 + 5X}.$$

Solution to Exercise 3.3.

We have

$$\mu^{-1} = \frac{3}{4} Y + \frac{7}{14} Z, \qquad Y = Z = \frac{1}{2},$$

$$\mu^{-1} = \frac{3}{4} Y + \frac{7}{14} Z, \qquad Y = Z = \frac{1}{2}, \mu^{-1} = \frac{3}{4} Y + \frac{7}{14} Z, \qquad Y = Z = \frac{1}{2},$$

Hence

$$\mu^{-1} = \frac{3}{8} + \frac{7}{28} = \frac{3}{8} + \frac{1}{4} = \frac{5}{8}, \qquad \mu = \frac{8}{5} = 1.6.$$