Solution to Exercise 2.1.

We have

$$
\begin{aligned}
m_{app} &= -2.5 \log l + K_1 \\
&= -2.5 \log L + 2.5 \log \left(4\pi d^2\right) + K_1 \\
&= M_{abs} - K_2 + 2.5 \log (4\pi) + 5 \log d + K_1,
\end{aligned}
$$

$$
\begin{aligned}
m_{app} &= -2.5 \log l + K_1 \\
&= -2.5 \log L + 2.5 \log \left(4\pi d^2\right) + K_1 \\
&= M_{abs} - K_2 + 2.5 \log (4\pi) + 5 \log d + K_1,
\end{aligned}
$$

$$
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m_{app} &= -2.5 \log l + K_1 \\
&= -2.5 \log L + 2.5 \log \left(4\pi d^2\right) + K_1 \\
&= M_{abs} - K_2 + 2.5 \log (4\pi) + 5 \log d + K_1,
\end{aligned}
$$

which gives the required relation (2.8) when we choose

$$K_2 = K_1 + 2.5 \log (4\pi) + 5.$$

Solution to Exercise 2.2.

We have

$$l_B = \frac{L_B}{4\pi d^2}, \qquad l_U = \frac{L_U}{4\pi d^2},$$

$$l_B = \frac{L_B}{4\pi d^2}, \qquad l_U = \frac{L_U}{4\pi d^2},$$

$$l_B = \frac{L_B}{4\pi d^2}, \qquad l_U = \frac{L_U}{4\pi d^2},$$

$$\frac{l_B}{l_U} = \frac{L_B}{L_U},$$

$$\frac{l_B}{l_U} = \frac{L_B}{L_U},\frac{l_B}{l_U} = \frac{L_B}{L_U},$$

And hence $U - B$ $U - B U - B$ is independent on distance d $d d$; the same with other colour indices.

Solution to Exercise 2.3.

We introduce

$$x = \frac{hc}{\lambda kT}$$

$$x = \frac{hc}{\lambda kT} x = \frac{hc}{\lambda kT}$$

as new independent variable, which gives

$$
\begin{aligned}
I_{bol} &= \int_0^\infty I_\lambda d\lambda \\
&= 2\pi hc^2 \left(\frac{kT}{hc}\right)^4 \left(\frac{R}{d}\right)^2 \int_0^\infty \frac{x^3}{e^x - 1} dx \\
&= \sigma T^4 \left(\frac{R}{d}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
I_{bol} &= \int_0^\infty I_\lambda d\lambda \\
&= 2\pi hc^2 \left(\frac{kT}{hc}\right)^4 \left(\frac{R}{d}\right)^2 \int_0^\infty \frac{x^3}{e^x - 1} dx \\
&= \sigma T^4 \left(\frac{R}{d}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
I_{bol} &= \int_0^\infty I_\lambda d\lambda \\
&= 2\pi hc^2 \left(\frac{kT}{hc}\right)^4 \left(\frac{R}{d}\right)^2 \int_0^\infty \frac{x^3}{e^x - 1} dx \\
&= \sigma T^4 \left(\frac{R}{d}\right)^2
\end{aligned}
$$

where

$$
\sigma = 2\pi hc^2 \left(\frac{k}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} dx.
$$

$$
\sigma = 2\pi hc^2 \left(\frac{k}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} dx. \sigma = 2\pi hc^2 \left(\frac{k}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} dx.
$$

Solution to Exercise 2.4.

Parallax

$$p'' = \frac{1\ \mathrm{pc}}{d(\mathrm{pc})} = \frac{3.1 \times 10^{16}\,\mathrm{m}}{1.2 \times 10^{17}\,\mathrm{m}} = 0.258$$

$$p'' = \frac{1\ \mathrm{pc}}{d(\mathrm{pc})} = \frac{3.1 \times 10^{16}\,\mathrm{m}}{1.2 \times 10^{17}\,\mathrm{m}} = 0.258 \quad p'' = \frac{1\ \mathrm{pc}}{d(\mathrm{pc})} = \frac{3.1 \times 10^{16}\,\mathrm{m}}{1.2 \times 10^{17}\,\mathrm{m}} = 0.258$$

Distance

$$d(\mathrm{pc}) = \frac{1\ \mathrm{pc}}{p''} = 2\ \mathrm{pc} = 6.2 \times 10^{16}\,\mathrm{m}$$

$$d(\mathrm{pc}) = \frac{1\ \mathrm{pc}}{p''} = 2\ \mathrm{pc} = 6.2 \times 10^{16}\,\mathrm{m} \quad d(\mathrm{pc}) = \frac{1\ \mathrm{pc}}{p''} = 2\ \mathrm{pc} = 6.2 \times 10^{16}\,\mathrm{m}$$

True ratio of brightness is

$$10 \times \left(\frac{6.2}{12}\right)^2 = 2.67\,.$$

$$10 \times \left(\frac{6.2}{12}\right)^2 = 2.67 \quad 10 \times \left(\frac{6.2}{12}\right)^2 = 2.67\,.$$

Solution to Exercise 2.5.

a)

$$\begin{aligned} M_{abs} &= m_{app} - 5 \log d(\mathrm{pc}) + 5 \\ &= 7.62 - 10 + 5 = 2.62 \end{aligned}$$

$$\begin{aligned} M_{abs} &= m_{app} - 5 \log d(\mathrm{pc}) + 5 \quad M_{abs} = m_{app} - 5 \log d(\mathrm{pc}) + 5 \\ &= 7.62 - 10 + 5 = 2.62 \qquad\quad = 7.62 - 10 + 5 = 2.62 \end{aligned}$$

b) We have an equation

$$M_{abs} - M_{abs,\odot} = -2.5\log\frac{L}{L_\odot} = 2.62 - 4.62 = -2.0$$

$$M_{abs} - M_{abs,\odot} = -2.5\log\frac{L}{L_\odot} = 2.62 - 4.62 = -2.00$$

$$M_{abs} - M_{abs,\odot} = -2.5\log\frac{L}{L_\odot} = 2.62 - 4.62 = -2.00$$

from which

$$\log\frac{L}{L_\odot} = 0.8, \qquad \frac{L}{L_\odot} = 10^{0.8} = 6.3,$$

$$\log\frac{L}{L_\odot} = 0.8, \qquad \frac{L}{L_\odot} = 10^{0.8} = 6.3, \log\frac{L}{L_\odot} = 0.8, \qquad \frac{L}{L_\odot} = 10^{0.8} = 6.3,$$

$$L = 6.3 \times 3.86 \times 10^{26}\,\mathrm{J/s} = 2.43 \times 10^{27}\,\mathrm{J/s}$$

$$L = 6.3 \times 3.86 \times 10^{26}\,\mathrm{J/s} = 2.43 \times 10^{27}\,\mathrm{J/s}$$

$$L = 6.3 \times 3.86 \times 10^{26}\,\mathrm{J/s} = 2.43 \times 10^{27}\,\mathrm{J/s}$$

c)

$$R^2 = \frac{L}{4\pi\sigma T_{eff}^{\,4}} = \frac{2.43 \times 10^{27}}{4\pi \times 5.67 \times 10^{-8} \times 5780^4}$$

$$= 3.06 \times 10^{18}\,(\mathrm{m}^2), \qquad R = 1.75 \times 10^{9}\,\mathrm{m}$$

$$R^2 = \frac{L}{4\pi\sigma T_{eff}^{\ 4}} = \frac{2.43 \times 10^{27}}{4\pi \times 5.67 \times 10^{-8} \times 5780^4}$$

$$= 3.06 \times 10^{18} (m^2), \quad R = 1.75 \times 10^9 m$$

$$R^2 = \frac{L}{4\pi\sigma T_{eff}^{\ 4}} = \frac{2.43 \times 10^{27}}{4\pi \times 5.67 \times 10^{-8} \times 5780^4}$$

$$= 3.06 \times 10^{18} (m^2), \quad R = 1.75 \times 10^9 m$$

Second star:

$$L_2 = 100^2 L_1,$$

$$M_{abs,2} = M_{abs,1} - 2.5 \log \frac{L_2}{L_1} = 2.62 - 10 = -7.38,$$

$$m_{app,2} = M_{abs,2} + 5 \log d_2 - 5 = 13.12,$$

$$5 \log d_2 = 13.12 + 5 + 7.38 = 25.5,$$

$$\log d_2 = 5.1, \qquad d_2 = 10^{5.1} = 1.25 \times 10^5 (pc),$$

$$p = 8.0 \times 10^{-6} \text{arc sec}$$

Solution to Exercise 2.6.

$$m_{app} = -2.5 \log (count) + const$$

$$m_{app} = -2.5 \log (count) + const \quad m_{app} = -2.5 \log (count) + const$$

In R filter:

$$m_{1R} - m_{2R} = -2.5 \log \frac{23456}{58919} = 1.000,$$

$$m_{2R} = m_{1R} - 1.0 = 12.1 - 1.0 = 11.1$$

$$m_{1R} - m_{2R} = -2.5 \log \frac{23456}{58919} = 1.000,$$

$$m_{2R} = m_{1R} - 1.0 = 12.1 - 1.0 = 11.1$$

$$m_{1R} - m_{2R} = -2.5 \log \frac{23456}{58919} = 1.000,$$

$$m_{2R} = m_{1R} - 1.0 = 12.1 - 1.0 = 11.1$$

In B filter:

$$m_{1B} - m_{2B} = -2.5 \log \frac{20954}{49405} = 0.932$$

$$m_{1B} - m_{2B} = -2.5 \log \frac{20954}{49405} = 0.932$$

$$m_{1B} - m_{2B} = -2.5 \log \frac{20954}{49405} = 0.932$$

For solar-type star,

$$B_1 - R_1 = 1.17$$

$$B_1 - R_1 = 1.17$$

$$B_1 - R_1 = 1.17$$

(same as for the Sun), hence

$$B_1 \equiv m_{1B} = 1.17 + 12.1 = 13.27$$

$$B_1 \equiv m_{1B} = 1.17 + 12.1 = 13.27$$

$$B_1 \equiv m_{1B} = 1.17 + 12.1 = 13.27$$

For other star,

$$B_2 \equiv m_{2B} = m_{1B} - 0.932 = 12.34,$$
$$B_2 - R_2 \equiv m_{2B} - m_{2R} = 12.34 - 11.1 = 1.24$$

$$B_2 \equiv m_{2B} = m_{1B} - 0.932 = 12.34,$$
$$B_2 - R_2 \equiv m_{2B} - m_{2R} = 12.34 - 11.1 = 1.24$$
$$B_2 \equiv m_{2B} = m_{1B} - 0.932 = 12.34,$$
$$B_2 - R_2 \equiv m_{2B} - m_{2R} = 12.34 - 11.1 = 1.24$$

For the second star, $B - R$ $B - R$$B - R$ is bigger, hence it is redder.

Solution to Exercise 2.7.

a)

$$M_{abs} = 4.62 - 2.5 \log \frac{L}{L_\odot}$$

$$M_{abs} = 4.62 - 2.5 \log \frac{L}{L_\odot}$$

$$M_{abs} = 4.62 - 2.5 \log \frac{L}{L_\odot}$$

Reigel:

$$-6.77 = 4.62 - 2.5 \log \frac{L_1}{L_\odot},$$

$$\log \frac{L_1}{L_\odot} = \frac{11.39}{2.5} = 4.56, \qquad \frac{L_1}{L_\odot} = 3.6 \times 10^4$$

$$-6.77 = 4.62 - 2.5 \log \frac{L_1}{L_\odot},$$

$$\log \frac{L_1}{L_\odot} = \frac{11.39}{2.5} = 4.56, \qquad \frac{L_1}{L_\odot} = 3.6 \times 10^4$$

$$-6.77 = 4.62 - 2.5 \log \frac{L_1}{L_\odot},$$

$$\log \frac{L_1}{L_\odot} = \frac{11.39}{2.5} = 4.56, \qquad \frac{L_1}{L_\odot} = 3.6 \times 10^4$$

β Canis Majoris:

$$-1.33 = 4.62 - 2.5 \log \frac{L_2}{L_\odot},$$

$$\log \frac{L_2}{L_\odot} = 2.38, \qquad \frac{L_2}{L_\odot} = 240$$

$$-1.33 = 4.62 - 2.5 \log \frac{L_2}{L_\odot},$$

$$\log \frac{L_2}{L_\odot} = 2.38, \qquad \frac{L_2}{L_\odot} = 240$$

$$-1.33 = 4.62 - 2.5 \log \frac{L_2}{L_\odot},$$

$$\log \frac{L_2}{L_\odot} = 2.38, \qquad \frac{L_2}{L_\odot} = 240$$

b)

$$\frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2 ,$$

$$\frac{3.6 \times 10^4}{240} = \left(\frac{R_1}{R_2}\right)^2 , \quad \frac{R_1}{R_2} = 12 .$$

$$\frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2 , \qquad \frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2 ,$$

$$\frac{3.6 \times 10^4}{240} = \left(\frac{R_1}{R_2}\right)^2 , \quad \frac{R_1}{R_2} = 12 . \quad \frac{3.6 \times 10^4}{240} = \left(\frac{R_1}{R_2}\right)^2 , \quad \frac{R_1}{R_2} = 12 .$$

Solution to Exercise 2.8.

We have

$$L \propto T^6$$

$$L \propto T^6 L \propto T^6$$

And also

$$L \propto R^2 T^4 ,$$

$$L \propto R^2 T^4 , L \propto R^2 T^4 ,$$

hence

$$T^6 \propto R^2 T^4 , \quad T^2 \propto R^2 , \quad T \propto R .$$